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The effect of a strong magnetic field on the binding energy and the photoionization cross-section in a quantum well

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Abstract. The effect of a strong magnetic field on the binding energy and the photon energy dependence of the photoionization cross-section as a function of the well size is studied in a $GaAs/Ga_{1-x}Al_xAs$ quantum well for several values of the magnetic field, taking into account the finite character of the barrier potential. The results we have obtained show that the applied strong magnetic field affects drastically the binding energy and the photoionization cross-section and this effect is more significant in a quasi-two-dimensional structure than in a three-dimensional system.

1. Introduction

The problem of an electron bound to an impurity atom plunged in a magnetic field plays a fundamental role in understanding the optical properties of impurities in semiconductors. In this work, we concentrate on one of the most interesting optical properties, that is the photoionization cross-section. The latter is needed for the characterization of impurity states in semiconductors and particularly in the low-dimensional electronic systems such as quantum well structures. Various calculations incorporating the magnetic field effect have described the impurity states essentially in two regions: the high-field region and the low-field region. The high-field region which is important in many problems for astrophysicists and for solid-state physicists and where the magnetic energy is large compared with the Coulomb energy, requires a multiband non-parabolic effect, especially in semiconductors with a narrow gap like InSb. In the low-field region where the Coulomb potential term dominates over the magnetic field one, the parabolic models are useful for the description of the low-field donor states. The study of the energy levels as well as polaronic effects on the levels of a hydrogenic impurity atom incorporating the magnetic field effect in quantum wells has been extensively reported in the past [1–16]. In contrast to the photoionization process in the bulk case where the electronic final state remains the same for the three photon polarization directions, the photoionization cross-section spectral in the low-dimensional electronic systems such as quantum wells (QWs), quantum-well wires and quantum dots, depends, besides the type of impurity wavefunction and the potential which binds the charge carrier to the impurity centre and the band structure of the host crystal, on the nature of the wavefunction of the subband into which the charge carrier (electron or hole) is excited. This is due to the additional electron confinement achieved by the reduction of the dimensionality in these microstructures. In recent years, some theoretical and experimental studies of the photoionization cross-section in $GaAs/Ga_{1-x}Al_xAs$ quantum wells have been reported. Takikawa *et al* [17] have both experimentally and theoretically

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determined the photoionization threshold energy and the excitation energy dependence of the photoionization cross-section of deep traps in AlGaAs/GaAs quantum well layers by metallorganic chemical-vapour deposition. El-Said and Tomak [18, 19] have calculated the dependence of photoionization cross-section on photon energy for excitation of a shallow bound electron from the impurity ground state to the conduction subbands in a $GaAs/Ga_{1-x}Al_xAs$ quantum well by considering the photon polarization direction along the perpendicular to the electron confinement direction. In their photoionization calculations, they considered the simplest case of an infinite confining potential. Recently, Ilaiwi and El-Said [20] improved their calculations by using a finite potential barrier at the interfaces in the case of z-polarized photoionizing light. More recently, Sali *et al* [21] have evaluated the photon energy dependence of the photoionization cross-section and the binding energy of an isolated hydrogenic donor impurity in infinite-barrier II–VI quantum well structures as a function of well width, taking into account the interaction between electron and bulk longitudinal-optical phonons.

In quantum-well wire structures where free motion is possible only along the length of the wire, some theoretical studies have been advocated, concentrating on the spectral dependence of the photoionization cross-section as a function of photon energy of a hydrogenic donor impurity [22, 23].

In a previous work, Sali *et al* [24] determined the effect of a strong magnetic field on the photoionization cross-section of a hydrogenic shallow donor impurity in a three-dimensional (3D) system. They have found that the impurity photoionization is very sensitive to that applied of a strong magnetic field. It is expected that these results will be more pronounced as the electronic confinement is increased with the reduction of the dimensionality and this is rather encouraging for studies of the photoionization of impurities plunged into a strong magnetic field in confined microelectronic systems such as quantum well.

In this paper we will study the effect of a strong magnetic field B on both the binding energy and the photoionization cross-section for excitation of a shallow bound electron from the impurity ground state to a first conduction subband in a $GaAs/Ga_{1-x}Al_xAs$ quantum well. The potential barrier height in the $Ga_{1-x}Al_xAs$ region is taken to be finite and the external magnetic field is applied along the z direction, parallel to the well growth axis and perpendicular to the interfaces of the quantum well structure. Calculations of the photoionization cross-sections are performed for the electric field polarization of the photon perpendicular to the applied magnetic field. In our calculations, we neglect the contribution arising from the differences in the effective masses and the dielectric constants in the two semiconductors, since Greene and Bajaj [25] have shown that the difference between calculations with and without taking into consideration these contributions is very small. In section 2, we describe the theory of the photoionization cross-section, which leads to an expression of the photoionization cross-section valid in the absence and presence of a strong magnetic field in a finite quantum well. In section 3, we present the numerical results, and discussions as well as some concluding remarks that will prove to be important.

2. Photoionization cross-section theory

Under the action of electromagnetic radiation, the transition of an electron residing in the ground state of a donor impurity atom to the conduction subband continuum state requires sufficient energy in order to occur. In the presence of an applied magnetic field heading in the z direction, this process of photoexcitation to the first conduction subband state occurs only if the photon energy $\hbar\omega$ is larger than $\gamma + 2E_1 - E_{min}$ where $\gamma = \varepsilon_0^2\hbar^3B/(m^2ce^3)$ is the effective magnetic field parameter which measures the strength of the magnetic field B and ε_0 and m are the static dielectric constant and the electronic effective mass, respectively. E_1 is the energy of the first conduction subband without the impurity potential and E_{min} is the minimized energy of the impurity ground state (the expression of E_1 and E_{min} will be seen later). Hence, the photoionization cross-section for transition between an initial ground state $|\psi_i\rangle$ and a conduction subband final state $|\psi_f\rangle$ in the well known dipole approximation is given by:

$$\sigma(\hbar\omega) = \left[\left(\frac{\zeta_{eff}}{\zeta_0}\right)^2 \frac{n_r}{\varepsilon}\right] \frac{4\pi^2}{3} \alpha_{FS} \hbar\omega \sum_f |\langle\psi_i|r|\psi_f\rangle|^2 \delta(E_f + \gamma + E_1 - E_{min} - \hbar\omega) \tag{1}$$

where n_r is the optical index of refraction, ε is the dielectric constant of the medium, α_{FS} is the fine structure constant $e^2/\hbar c$ and E_f is the energy of the final state. $(\zeta_{eff}/\zeta_0)^2$ is a rate multiplication factor where ζ_{eff} is the effective electric field at the centre and ζ_0 is the average field [26] and $\langle\psi_i|r|\psi_f\rangle$ is the position matrix element between the impurity and the subband states.

From the expression (1), it appears that the calculation of the photoionization cross-section requires the evaluation of the position matrix element between the impurity and the conduction subband continuum states. This needs the knowledge of the initial ground state and final sate wave functions.

The Hamiltonian of a system consisting of an electron bound to a donor ion, inside a quantum well of width L, with finite potential barrier at the interfaces, in the presence of an applied magnetic field B can be written as [27]

$$H = -(\nabla_x^2 + \nabla_y^2 + \nabla_z^2) - \frac{2}{r} + V(z) + \gamma L_z + \frac{\gamma^2}{4}(x^2 + y^2) \tag{2}$$

where $V(z)$ is the finite potential barrier of the quantum well defined as

$$V(z) = \begin{cases} 0 & \text{for } |z| < L/2 \\ V_0 & \text{for } |z| > L/2. \end{cases} \tag{3}$$

The Hamiltonian equation (2) is written in units of energy $R^* = me^4/(2\varepsilon_0^2\hbar^2)$ (the bulk GaAs effective Rydberg) and length $a^* = \varepsilon_0\hbar^2/(me^2)$ (effective Bohr radius in GaAs). L_z is the z component of the orbital angular momentum.

For a $GaAs/Ga_{1-x}Al_xAs$ system, the conduction band discontinuity V_0 is assumed to be 85% of the total band-gap difference between the two semiconductors. The direct band-gap difference between the barrier and the well is given by the empirical formula [28] $\Delta E_g = 1.115x + 0.37x^2$ eV.

The presence of the Coulomb term in the Hamiltonian equation (2) leads to a non-separable differential equation which cannot be solved analytically and therefore there are only approximate methods such as the variational approach for determining the eigenstates of the Hamiltonian, especially for the ground state. Taking into account the compression of the impurity atom in the transverse dimensions by the application of a strong magnetic field, the finite character of the barrier potential and the hydrogenic impurity potential, we choose for the impurity ground state a normalized trial wavefunction with two variational parameters, which can be written as

$$\psi_i(r) = N_1 \varphi_1(z) \exp\left[-\left(\frac{x^2 + y^2}{4a_t^2} + \frac{z^2}{4a_1^2}\right)\right] \tag{4}$$

where N_1 is a normalization constant given by

$$N_1 = \left\{2\pi a_t^2 \left[I\left(\alpha_1, \frac{1}{2a_1^2}\right) + \sqrt{2\pi a_1}\cos^2\left(\alpha_1 \frac{L}{2}\right)\exp(\beta_1 L + 2\beta_1^2 a_1^2) \right.\right.$$
$$\left.\left. \times \operatorname{erfc}\left[\frac{L}{2\sqrt{2}a_1} + \sqrt{2}\beta_1 a_1\right]\right]\right\}^{-1/2} \tag{5}$$

and

$$I(\alpha,\beta) = \frac{1}{2}\sqrt{\frac{\pi}{\beta}}\left\{\mathrm{erf}\left(\frac{L}{2}\sqrt{\beta}\right) + \frac{1}{2}\exp\left(-\frac{\alpha^2}{\beta}\right)\right.$$
$$\left.\times\mathrm{Re}\left[\mathrm{erf}\left(\frac{L}{2}\sqrt{\beta} - \mathrm{i}\frac{\alpha}{\sqrt{\beta}}\right) + \mathrm{erf}\left(\frac{L}{2}\sqrt{\beta} + \mathrm{i}\frac{\alpha}{\sqrt{\beta}}\right)\right]\right\} \tag{6}$$

in which erf and erfc denote, respectively, the error function and the complementary error function.

The wavefunction $\varphi_1(z)$ of the square-well potential associated with the Hamiltonian (2) without the impurity potential and magnetic field is given by

$$\phi_1(z) = \begin{cases} \cos\left(\alpha_1\frac{L}{2}\right)\exp(\beta_1 L/2)\exp(-\beta_1|z|) & |z| > L/2 \\ \cos(\alpha_1 z) & |z| < L/2. \end{cases} \tag{7}$$

The exponential term in (4) associated with the Hamiltonian (2) without $V(z)$ describes the impurity ground state in a very strong magnetic field [29]. a_t and a_1 are the variational parameters which may be thought of as the effective Bohr radii of the donor ground state.

In equation (7), the parameters α_1 and β_1 are determined from the first subband energy by

$$\alpha_1 = (2mE_1/\hbar^2)^{1/2} \text{ and } \beta_1 = [2m(V_0 - E_1)/\hbar^2]^{1/2}. \tag{8}$$

The lowest subband energy E_1 of the square-well potential is determined numerically by requiring continuity of the derivative of the wavefunction at the interface, which yields

$$\left(\frac{E_1}{V_0}\right)^{1/2} = \cos\left(\frac{mE_1}{\hbar^2}\frac{L^2}{2}\right)^{1/2}. \tag{9}$$

For the evaluation of the Coulomb energy $E_{cou} = \langle\psi_i|-2/r|\psi_i\rangle$, the impurity potential matrix element is a three-dimensional integral and we have reduced it only to a one-dimensional form by using the following representation of the Coulomb potential

$$\frac{1}{\sqrt{\rho^2+z^2}} = \int_0^\infty \exp(-zt)J_0(\rho t)\,\mathrm{d}t \qquad \text{for } z > 0 \tag{10}$$

where J_0 is the zeroth-order spherical Bessel function of the first kind.

The expectation value $E = \langle\psi_i|H|\psi_i\rangle$ in the presence of an applied magnetic field in a finite well leads to:

$$E = \frac{1}{2a_t^2}\left(1+\frac{\mu^2}{2}\right) + \frac{\gamma^2 a_t^2}{2} + \sqrt{2}\pi^{3/2}\alpha_1^2 a_t^2 a_1 N_1^2 \mathrm{erf}\left(\frac{L}{2\sqrt{2}a_1}\right) + E_p + E_{cou} \tag{11}$$

where

$$E_p = (2\pi)^{3/2}a_t^2 a_1 N_1^2 V_0 \cos^2\left(\alpha_1\frac{L}{2}\right)\exp(\beta_1 L + 2\beta_1^2 a_1^2)\mathrm{erfc}\left(\frac{L}{2\sqrt{2}a_1} + \sqrt{2}\beta_1 a_1\right)$$

and

$$E_{cou} = -2(2\pi)^{3/2}a_t N_1^2\left[\int_0^{L/2}\cos^2(\alpha_1 z)\exp\left(-\frac{z^2}{2a_t^2}(\mu^2-1)\right)\mathrm{erfc}\left(\frac{z}{a_t\sqrt{2}}\right)\mathrm{d}z\right.$$
$$+\cos^2\left(\alpha_1\frac{L}{2}\right)\exp(\beta_1 L)$$
$$\left.\times\int_{L/2}^\infty \exp(-2\beta_1 z)\exp\left(-\frac{z^2}{2a_t^2}(\mu^2-1)\right)\mathrm{erfc}\left(\frac{z}{a_t\sqrt{2}}\right)\mathrm{d}z\right]$$

where $\mu = a_t/a_1$ is the ratio between the transverse and the longitudinal effective Bohr radii.

The donor binding energy E_b of the ground state is obtained by subtracting the minimized energy $E_{min} = \min\langle\psi_i|H|\psi_i\rangle$ from the lowest subband energy E_1 and the energy of the first Landau level γ [30], i.e. $E_b = E_1 + \gamma - E_{min}$.

For the photoionization process, the electronic final state of the conduction subband into which the electron is emitted depends on the light polarization direction. This is to be contrasted with the bulk case where the electronic final state remains the same for the three directions of the incident light polarization. For light polarized in the x direction and by neglecting the effect of the impurity potential on the conduction subband continuum state, the first allowed dipole transition that take place from the impurity ground state to the first subband continuum state is described by the following photoexcited electron wavefunction.

$$\psi_f(\rho, z, k_\perp) = \frac{1}{\sqrt{S}}\phi_1(z)\exp(\mathrm{i}k_\perp \cdot \rho) \tag{12}$$

where

$$\phi_1(z) = \begin{cases} N_f \cos(\alpha_1 L/2)\exp(\beta_1 L/2)\exp(-\beta_1|z|) & |z| > L/2 \\ N_f \cos(\alpha_1 z) & |z| < L/2 \end{cases} \tag{13}$$

and N_f is a normalization constant of $\phi_1(z)$ and is given by

$$N_f = \left[\frac{L}{2} + \frac{\sin(\alpha_1 L)}{2\alpha_1} + \frac{\cos^2(\alpha_1 L/2)}{\beta_1}\right]^{-1/2}. \tag{14}$$

By inserting equations (4) and (12) into (1) and by transforming the summation over the final state into a two-dimensional integral, we obtain for the photon energy dependence of the photoionization cross-section associated with a shallow donor impurity–first conduction subband transition in the presence of a strong magnetic field, taking into account the finite character of the barrier potential and for photon energies above the photoionization threshold energy E_s,

$$\begin{aligned}\sigma(\hbar\omega) = {} & \left[\left(\frac{\zeta_{eff}}{\zeta_0}\right)^2 \frac{n_r}{\varepsilon}\right] 32\pi^3 \alpha_{FS} N_1^2 N_f^2 a_t^8 R^2(\alpha_1, \beta_1)\left(\frac{2m}{\hbar^2}E_s\right)^2 S(S-1) \\ & \times \exp\left(-2\left(\frac{2m}{\hbar^2}\right) a_t^2 E_s (S-1)\right)\end{aligned} \tag{15}$$

where

$$S = \hbar\omega/E_s$$

$$E_s = E_b + \frac{\hbar^2}{2m}N_f^2\frac{L}{2}\alpha_1^2 + \frac{V_0}{\beta_1}N_f^2\cos^2(\alpha_1 L/2)$$

$$R(\alpha_1, \beta_1) = I\left(\alpha_1, \frac{1}{4a_1^2}\right) + 2\sqrt{\pi}a_1\cos^2(\alpha_1 L/2)\exp(\beta_1 L + 4a_1^2\beta_1^2)\mathrm{erfc}\left(\frac{L}{4a_1} + 2\beta_1 a_1\right)$$

and the other parameters are defined as before.

The resulting expression (15) for the photoionization cross-section can be evaluated numerically.

3. Results and discussions

The photoionization cross-section formula equation (15) associated with an excitation of a shallow bound electron from the impurity ground state to a first conduction subband in a finite well model can now be evaluated numerically for different values of the effective magnetic field parameter γ. For numerical computations, we use the GaAs–$Ga_{1-x}Al_x$As quantum well to evaluate the photoionization cross-section of a shallow-doped donor impurity located at the centre of the quantum well. Since $GaAs/Ga_{1-x}Al_xAs$ is the simplest, the most extensively studied and in which almost all the material properties are well known. However, in order to calculate the photoionization cross-section, it is necessary to know the effective field parameter ζ_{eff} at the impurity site which is fairly difficult to calculate. The photoionization cross-section has, therefore, been calculated by evaluating the effective field ratio ζ_{eff}/ζ_0. The latter, has, generally, been evaluated by the adjustment of the theoretical cross-section to the experimental one and, generally, it may be small for weakly localized electrons and large for strongly localized electrons. As is well known, this factor does not affect the shape of the cross-section and according to our knowledge, unfortunately, no experimental data on the photoionization of the shallow impurity level in quantum well are available, and a comparison with experiment is not yet possible. In what follows, we therefore assume that the effective field ratio ζ_{eff}/ζ_0 is equal approximately to unity.

The values of the physical parameters pertaining to the $GaAs/Ga_{1-x}Al_xAs$ system in the calculation of the binding energy and the photoionization cross-section are: $\varepsilon_0 = 12.5$ and $m = 0.067m_0$ where m_0 is the free electron mass. The variational parameters a_t and a_1 which minimize the expectation value of the Hamiltonian of the system were calculated as a function of the size of the quantum well for several values of the barrier height of the confining potential for different values of the effective magnetic field parameter γ. We have restricted our calculations to values of γ such as $\gamma \leqslant 10$ for which the effect of a multiband non-parabolic effect is negligible. With the optimum parameters we evaluate the impurity binding energies and the photoionization cross-sections.

In figure 1 we display the variation of the binding energy of the ground state of a shallow donor impurity as a function of GaAs well size for $V_0 = 58.134R^*$ and $79.78R^*$ corresponding

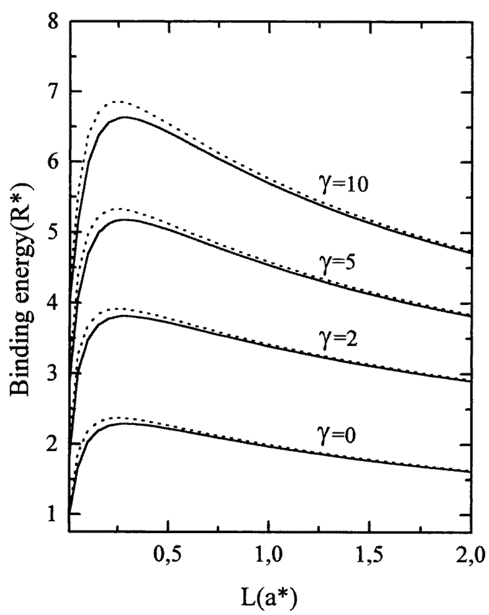

Figure 1. Variation of the binding energy of the ground state for a donor at the centre of a GaAs quantum well as a function of the well size (L) for different values of the magnetic field parameter γ and for two values of the Al concentration of $Ga_{1-x}Al_xAs$, $x = 0.4$ (dotted line) and $x = 0.3$ (solid line).

to the Al concentrations $x = 0.30$ and $x = 0.4$, respectively, and for a few selected values of γ. As expected, for a given value of the fraction x of the aluminium and a given value of γ, the binding energy increases as the size of the well is reduced until it reaches a maximum value, and then decays sharply to a value characteristic of bulk $Ga_{1-x}Al_xAs$ at $L = 0$. The same figure reveals that the difference between curves of different value of γ increases as the size of the well decreases. Since the shrinkage of the wavefunction in the x–y plane at smaller well widths decreases the diamagnetic term of the Hamiltonian equation (2) which is proportional to $(x^2 + y^2)$. This decrease in turn leads to an increase in the binding energy.

In order to show the validity of the impurity ground state envelope function that we worked with in the range of large magnetic fields, we compare the results of the energy levels obtained in this work with those obtained by Chen *et al* [13] using a novel variational method that is valid for the whole range of magnetic fields and for a well size $L < 300$ Å. The results of the energy levels and the binding energies obtained in this paper are compared in table 1 with those of Chen *et al* [13]. In our calculations we have used the same values of the physical parameters as those used by the latter authors. The binding energies in parenthesis are obtained by using the energy level results of Chen *et al* [13]. The general characteristic features and implications of the present calculations are in accordance with those obtained by Chen *et al* for strong magnetic fields. Even though the trial wavefunction we have used is unable to treat all the range of magnetic field strengths, it does, however, give a reasonable description of the system for relatively large magnetic fields.

Table 1. Energy levels E_{min} and binding energies E_b of confined impurity for a well width $L = 125$ Å as a function of the effective magnetic field parameter γ. The values in parenthesis are those of Chen *et al* [13].

γ	E_{min} (cm^{-1})	E_b (R^*)
1.0	108.61 (104.81)	2.65 (2.73)
1.2	112.10 (109.56)	2.78 (2.83)
1.4	115.95 (114.55)	2.89 (2.92)
1.6	120.17 (119.77)	3.00 (3.01)
1.8	124.71 (125.13)	3.11 (3.10)
2.0	129.51 (130.63)	3.20 (3.18)
2.2	134.56 (136.27)	3.29 (3.26)
2.4	139.82 (142.01)	3.38 (3.34)
2.6	145.27 (147.89)	3.47 (3.41)
2.8	150.89 (153.86)	3.55 (3.49)
3.0	156.68 (159.93)	3.62 (3.56)
3.2	162.60 (166.09)	3.70 (3.63)
3.4	168.66 (172.39)	3.77 (3.69)
3.6	174.83 (178.74)	3.84 (3.76)
3.8	181.12 (185.18)	3.90 (3.82)
4.0	187.51 (191.72)	3.97 (3.88)
4.2	194.00 (198.30)	4.03 (3.94)
4.4	200.57 (204.98)	4.09 (4.00)

In figure 2, we have plotted the photoionization cross-section as a function of the normalized photon energy $\hbar\omega/E_s$ in the absence of the magnetic field and for different values of the effective magnetic field parameter γ for a fixed quantum well size $L = a^*$ and for an alloy composition of $Ga_{1-x}Al_xAs$, $x = 0.40$. As we can note from this figure, as expected, the use of a plane wave in the x–y plane for the final state leads to zero absorption at the photoionization threshold energy. At zero magnetic field, the photoionization cross-section rises sharply from zero absorption, peaks at lower photon energies and then decays monotonically like the true

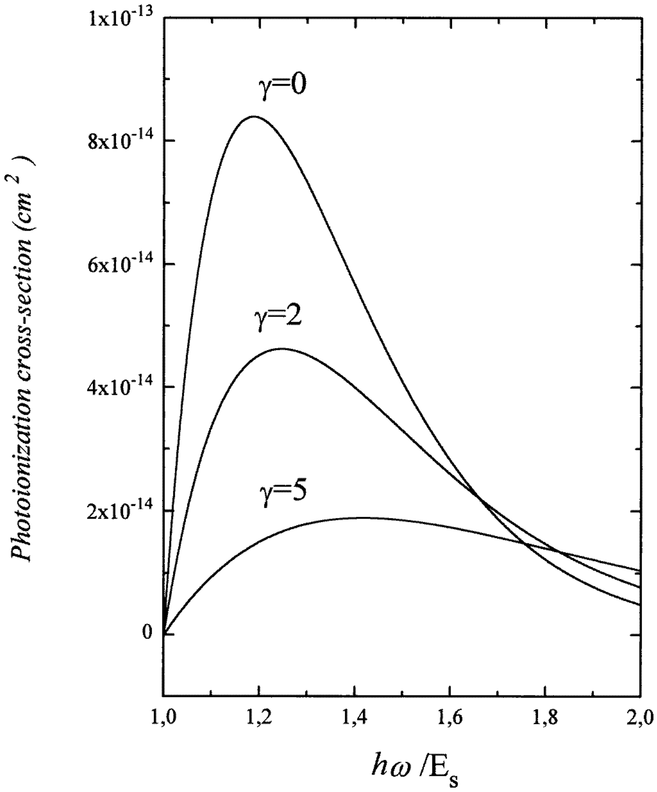

Figure 2. Variation of the photoionization cross-section as a function of the normalized photon energy $\hbar\omega/E_s$ for several values of the effective magnetic field parameter γ for a fixed well size $L = a^*$ and for the Al concentration of $Ga_{1-x}Al_xAs$, $x = 0.4$.

hydrogenic model [31] for higher photon energies. The peak at lower energies originates from the localized electron wavefunction near the edge of the small system. Since for wells of small size ($L = a^*$) and for weak magnetic field, the dominant contribution to the binding energy and as a consequence to the photoionization cross-section comes from the Coulomb and the confinement potential. For broad quantum wells, we would expect a peak value of the cross-section converging to the bulk value [24] which occurs at slightly larger photon energies.

As the effective magnetic field parameter γ increases, the magnitude of the cross-section becomes much smaller, the value of $\hbar\omega$ associated with the peak of the cross-section moves to higher photon energies and the photoionization cross-section decreases slowly for higher photon energies like the δ-function potential [32]. The reason for this behaviour is that for large values of the effective magnetic field parameter γ, the binding energy is relatively insensitive to the effect of the well confining potential except for very narrow wells which is not our case and the overall shape of the photoionization cross-section is sensitive to the magnetic field effect, since very high magnetic field confines the electron close the growth well axis, leading in effect to an increasing of the binding energies and hence to the optical photoionization threshold energy. As a consequence the excitation of an electron linked to a donor impurity to the first conduction subband by absorption of a photon requires higher photon energies in order for the transition to occur.

It is important to mention that the overall shape of the spectral dependence of the photoionization cross-section we have obtained for an incident light polarized in the x direction is thoroughly different for light polarized along the growth axis of the well [20] since, for the latter, the maximum absorption is found at the photoionization threshold and the photoionization cross-section decreases monotonically from the maximum with increasing photon energy.

It is interesting to note that the decrease of the magnitude of the cross-section which leads to an increase of the effective field parameter ζ_{eff} at the impurity centre and the increase of the shift of the optical photoionization threshold with increasing magnetic field are expected to be more pronounced in a thin quantum well than in a three-dimensional system [24]. This remarkable effect of the magnetic field is due simply to the increasing electronic confinement with the reduction of the dimensionality.

In this work, we have reported a calculation of the binding energies of the ground state of a hydrogenic donor associated with the first subband and the photoionization cross-section associated with a transition from the impurity ground state to the first conduction subband in a $GaAs/Ga_{1-x}Al_xAs$ quantum well in the presence of an applied strong magnetic field. The magnetic field is assumed to be parallel to the growth axis of the well. The calculations have been performed by using the variational method, choosing a Gaussian trial wavefunction with two variational parameters for the impurity ground state that takes into account the confinement of the carriers in a finite well, the compression of the impurity atom in the x–y plane by the application of an external strong magnetic field, and the electron–impurity Coulomb interaction. We have shown that the external strong magnetic field affects more drastically the binding energies and the donor impurity photoionization cross-section in a quasi-two-dimensional system than in a three-dimensional case. The results we have obtained plainly show the importance of a fully consistent treatment of the problem of a quasi-two-dimensional hydrogenic donor impurity plunged into a strong magnetic field in the calculations of the binding energy and the photoionization cross-section as a function of the photon energy, the size of the well and the magnetic field.

Takikawa *et al* [17] have experimentally studied the photoionization cross-section of a deep trap in $GaAs/Ga_{1-x}Al_xAs$ quantum well, while at present there are any reports on experimental values of the photoionization cross-section of a hydrogenic shallow donor impurity in quantum well structures. The measurement of this shallow impurity photoionization should be of great interest in understanding the optical properties of carriers in quantum well structures.

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